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## Electrical Engineering



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### **3500 Multiple Choice Questions for ESE, GATE, PSUs : Electrical Engineering**

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## PREFACE



**B. Singh** (Ex. IES)

It gives me great happiness to introduce the **Revised Edition** on Electrical Engineering containing nearly 3500 MCQs which focuses in-depth understanding of subjects at basic and advanced level which has been segregated topicwise to disseminate all kind of exposure to students in terms of quick learning and deep apt. The topicwise segregation has been done to align with contemporary competitive examination

pattern. Attempt has been made to bring out all kind of probable competitive questions for the aspirants preparing for ESE, GATE & PSUs. The content of this book ensures threshold level of learning and wide range of practice questions which is very much essential to boost the exam time confidence level and ultimately to succeed in all prestigious engineer's examinations. It has been ensured from MADE EASY team to have broad coverage of subjects at chapter level.

While preparing this book utmost care has been taken to cover all the chapters and variety of concepts which may be asked in the exams. The solutions and answers provided are upto the closest possible accuracy. The full efforts have been made by MADE EASY Team to provide error free solutions and explanations.

I have true desire to serve student community by way of providing good sources of study and quality guidance. I hope, this book will be proved an important tool to succeed in competitive examinations. Any suggestions from the readers for the improvement of this book are most welcome.

**B. Singh** (Ex. IES)

Chairman and Managing Director  
MADE EASY Group

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# UNIT 1

## Electromagnetic Theory

### 1. Vector Analysis

- Q.1** If  $\vec{P} = x^2y^2\vec{i} + (x-y)\vec{k}$ ,  $\vec{Q} = zx\vec{i}$  and  $\phi = xy^2z^3$ , then match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

**List-I**

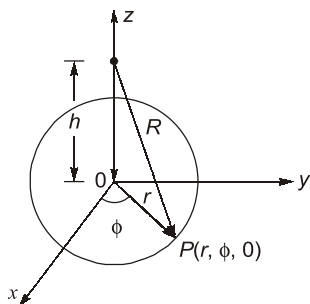
**List-II**

- |                   |  |
|-------------------|--|
| A. Div. $\vec{Q}$ | 1. $y^2z^3\vec{i} + 2yxz^3\vec{j} + 3z^2y^2x\vec{k}$ |
| B. Grad $\phi$    | 2. $-\vec{i} + \vec{k}x^2$                           |
| C. Curl $\vec{P}$ | 3. $z$   |

**Codes:**

- |     | A | B | C |
|-----|---|---|---|
| (a) | 1 | 2 | 3 |
| (b) | 2 | 1 | 3 |
| (c) | 3 | 1 | 2 |
| (d) | 3 | 2 | 1 |

- Q.2** The unit vector  $\vec{a}_R$  which points from  $z = h$  on the  $z$ -axis towards  $(r, \phi, 0)$  in cylindrical co-ordinates as shown below is given by



- |   |   |
|---|---|
| (a) $\frac{h\vec{a}_r - r\vec{a}_z}{\sqrt{r^2 + h^2}}$    | (b) $\frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$    |
| (c) $\frac{h\vec{a}_\phi - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ | (d) $\frac{r\vec{a}_z - h\vec{a}_\phi}{\sqrt{r^2 + h^2}}$ |

- Q.3** If the vector  $V$  given below is irrotational, then the values of  $a$ ,  $b$  and  $c$  will be respectively
- $$V = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$$

- (a)  $a = 4$ ,  $b = 2$  and  $c = -1$   
 (b)  $a = 2$ ,  $b = -1$  and  $c = 4$   
 (c)  $a = 4$ ,  $b = -1$  and  $c = 2$   
 (d)  $a = 2$ ,  $b = 4$  and  $c = -1$

- Q.4** Match **List-I (Vector Identities)** with **List-II (Equivalent expression)** and select the correct answer using the codes given below the lists:

**List-I**

- A.  $(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D})$   
 B.  $\vec{A} \times (\vec{B} \times \vec{C})$   
 C.  $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D})$

**List-II**

1.  $(\vec{A} \cdot \vec{C} \cdot \vec{D})\vec{B} - (\vec{B} \cdot \vec{C} \cdot \vec{D})\vec{A}$   
 2.  $[(\vec{A} \cdot \vec{C})(\vec{B} \cdot \vec{D}) - (\vec{A} \cdot \vec{D})(\vec{B} \cdot \vec{C})]$   
 3.  $(\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$

**Codes:**

- |     | A | B | C |
|-----|---|---|---|
| (a) | 1 | 3 | 2 |
| (b) | 3 | 1 | 2 |
| (c) | 2 | 1 | 3 |
| (d) | 2 | 3 | 1 |

- Q.5** The vector differential operator, Del( $\nabla$ ) in spherical co-ordinate system is given by

- (a)  $\nabla = \vec{a}_r \frac{\partial}{\partial r} + \vec{a}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \vec{a}_\phi \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}$   
 (b)  $\nabla = \vec{a}_r \frac{1}{r} \frac{\partial}{\partial r} + \vec{a}_\theta \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} + \vec{a}_\phi \frac{\partial}{\partial \phi}$   
 (c)  $\nabla = \vec{a}_r \frac{\partial}{\partial r} + \vec{a}_\theta \frac{\partial}{\partial \theta} + \vec{a}_\phi \frac{1}{r \cos \theta} \frac{\partial}{\partial \phi}$   
 (d)  $\nabla = \vec{a}_r \frac{1}{r} \frac{\partial}{\partial r} + \vec{a}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \vec{a}_\phi \frac{1}{r \cos \theta} \frac{\partial}{\partial \phi}$

- Q.6 Assertion (A):** Divergence of a vector function  $\vec{A}$  at each point gives the rate per unit volume at which the physical entity is issuing from that point.

**Reason (R):** If some physical entity is generated or absorbed within a certain region of the field, then that region is known as source or sink respectively and if there are no sources or sinks in the field, the net outflow of the incompressible physical entity over any part of the region is zero. However, the net outflow is said to be positive, if the total strength of the sources are greater than the total strength of sink and vice-versa.

- (a) Both A and R are true and R is a correct explanation of A.  
 (b) Both A and R are true but R is not a correct explanation of A.  
 (c) A is true but R is false.  
 (d) A is false but R is true.

**Q.7** Which of the following identity is not true?

- (a)  $\vec{A}(\vec{B} \cdot \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$   
 (b)  $\nabla \cdot (\nabla \times \vec{A}) = 0$   
 (c)  $\nabla \times \nabla \phi \neq 0$   
 (d) None of the above

**Q.8** The vector  $\vec{A}$  directed from  $(2, -4, 1)$  to  $(0, -2, 0)$  in Cartesian coordinates is given by

- (a)  $-2\vec{a}_x + 2\vec{a}_y + \vec{a}_z$  (b)  $-2\vec{a}_x + 2\vec{a}_y - \vec{a}_z$   
 (c)  $-\vec{a}_x - 2\vec{a}_y + 2\vec{a}_z$  (d)  $\vec{a}_x - 2\vec{a}_y - \vec{a}_z$

**Q.9** What is the value of  $\iint_s \vec{F} \cdot d\vec{s}$ , where  $\vec{F} = 4xz\vec{i}_1 - y^2\vec{i}_2 + yz\vec{i}_3$  ?

Here,  $s$  is the surface bounded by  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$  and  $\vec{i}_1, \vec{i}_2, \vec{i}_3$  are unit vectors along  $x, y$  and  $z$  axes respectively.

- (a)  $1/2$  (b)  $5/2$   
 (c)  $2$  (d)  $3/2$

**Q.10** The vector field given by

$$\vec{A} = yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z \text{ is}$$

- (a) rotational and solenoidal  
 (b) rotational but not solenoidal  
 (c) irrotational and solenoidal  
 (d) irrotational but not solenoidal

**Q.11** If  $\vec{A} = \frac{\vec{a}_x}{\sqrt{x^2 + y^2}}$ , then the value of  $\nabla \cdot \vec{A}$  at  $(2, 2, 0)$  will be

- (a)  $-0.0884$  (b)  $0.0264$   
 (c)  $-0.0356$  (d)  $0.0542$

**Q.12** If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ , then the value of  $\vec{i} \times (\vec{r} \times \vec{i}) + \vec{j} \times (\vec{r} \times \vec{j}) + \vec{k} \times (\vec{r} \times \vec{k})$  is

- (a)  $\vec{r}$  (b)  $2\vec{r}$   
 (c)  $3\vec{r}$  (d)  $6\vec{r}$

**Q.13** What is the value of constant  $b$  so that the vector

$$\vec{V} = (x + 3y)\vec{i} + (y - 2x)\vec{j} + (x + bz)\vec{k}$$

is solenoidal?

- (a)  $2$  (b)  $-1$   
 (c)  $3$  (d)  $-2$

**Q.14** Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

**List-I**

- A. Gauss's divergence theorem  
 B. Stroke's theorem  
 C. The divergence  
 D. The curl

**List-II**

1.  $\nabla \cdot \vec{A}$   
 2.  $\oint_L \vec{A} \cdot d\vec{l} = \iiint_s (\nabla \times \vec{A}) \cdot d\vec{s}$   
 3.  $\iiint_s \vec{A} \cdot d\vec{s} = \iiint_s \vec{A} \cdot \vec{e} d\vec{s}$  ( $\vec{e}$  - An unit vector)  
 4.  $\nabla \times \vec{A}$

**Codes:**

	A	B	C	D
(a)	3	2	4	1
(b)	2	3	1	4
(c)	3	2	1	4
(d)	2	3	4	1

**Q.15 Assertion (A):** Vector differential operator is a vector quantity and it signifies that certain operations of a differentiation are to be carried out on the scalar function following it.

**Reason (R):** Vector differential operator possesses properties similar to ordinary vectors.

- (a) Both A and R are true and R is a correct explanation of A.  
 (b) Both A and R are true but R is not a correct explanation of A.  
 (c) A is true but R is false.  
 (d) A is false but R is true.

**Q.16** Consider the following statements:

1. Divergence of a vector function  $\vec{A}$  at each point gives the rate per unit volume at which the physical entity is issuing from that point.
2. If a vector function  $\phi$  represents temperature, then  $\text{grad. } \phi$  or  $\nabla\phi$  will represent rate of change of temperature with distance.
3. The curl of a vector function  $A$  gives the measure of the angular velocity at every point of the vector field.

Which of the above statements is/are correct?

- (a) 2 and 3 only      (b) 1, 2 and 3  
(c) 1 and 3 only      (d) 2 only

**Q.17 Assertion (A):** The Gauss's divergence theorem permits us to express certain integrals by means of surface integrals.

**Reason (R):** Gauss's divergence theorem states that "the surface integral of the curl of a vector field taken over any surface  $s$  is equal to the line integral of the vector field around the closed periphery (contour) of the surface."

- (a) Both A and R are true and R is a correct explanation of A.  
(b) Both A and R are true but R is not a correct explanation of A.  
(c) A is true but R is false.  
(d) A is false but R is true.

**Q.18** Match **List-I (Physical quantities)** with **List-II (Dimensions)** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Electric potential	1. $MT^{-2}I^{-1}$
B. Magnetic flux	2. $ML^2T^{-3}I^{-1}$
C. Magnetic field intensity	3. $IL^{-1}$
D. Magnetic flux density	4. $ML^2T^{-2}I^{-1}$

**Codes:**

A	B	C	D
(a) 2	4	3	1
(b) 4	2	3	1
(c) 1	2	1	3
(d) 4	2	1	3

**Q.19** Which of the following statements is not true regarding vector algebra?

- (a) Dot product of like unit vector is unity.  
(b) Dot product of unlike unit vector is zero.

(c) Cross product of two like unit vectors is a third unit vector having positive sign for normal rotation and negative for reverse rotation.

(d) All the above statements are true.

**Q.20** A rigid body is rotating with an angular velocity of  $\omega$  where,  $\vec{\omega} = \omega_x\vec{i} + \omega_y\vec{j} + \omega_z\vec{k}$  and  $v$  is the line velocity. If  $\vec{r}$  is the position vector given by  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ , then the value of  $\text{curl } \vec{v}$  will be equal to

- (a)  $\frac{1}{2}\omega$       (b)  $\omega$   
(c)  $\frac{1}{3}\omega$       (d)  $2\omega$

**Q.21** If  $\vec{r} = x\vec{i}_x + y\vec{i}_y + z\vec{i}_z$ , then which of the following relation will hold true?

- (a)  $\nabla\vec{r} = 3$       (b)  $\nabla \times \vec{r} = 0$   
(c) Both (a) and (b)      (d) Neither (a) nor (b)

**Q.22** If  $\vec{E}$  is any vector field in cartesian co-ordinates system, then

- (a)  $\nabla \cdot (\nabla \times \vec{E}) = \nabla \times \nabla \times \vec{E} - \nabla^2 \vec{E}$   
(b)  $\text{Div. curl } \vec{E} = 0$   
(c)  $\nabla \cdot (\nabla \times \vec{E}) = \nabla^2 \vec{E} - \nabla \times \nabla \times \vec{E}$   
(d)  $\text{Div. curl } \vec{E} \neq 0$

**Q.23** If  $\vec{c} = \vec{a} \times \vec{b}$  and  $\vec{b} = \vec{a} \times \vec{c}$ , then

- (a)  $\vec{b} = 0$  and  $\vec{c} = 0$       (b) Only  $\vec{b} = 0$   
(c) Only  $\vec{c} = 0$       (d)  $\vec{b} \neq 0$  and  $\vec{c} \neq 0$

**Q.24** If  $S$  is any closed surface enclosing a volume  $V$  and  $\vec{A} = ax\vec{i} + by\vec{j} + cz\vec{k}$ , then the value of

$\iiint_S \vec{A} \cdot \hat{n} d\vec{S}$  ( $\hat{n}$  is a unit vector) will be equal to

- (a)  $\frac{1}{3}(a+b+c)V$       (b)  $(a-b-c)V$   
(c)  $\frac{1}{2}(a+b+c)V$       (d)  $(a+b+c)V$

**Q.25 Assertion (A):** The laplacian operator of a scalar function  $\phi$  can be defined as "Gradient of the divergence of the scalar  $\phi$ ".



**Reason (R):** Laplacian operator may be a “scalar laplacian” or a “vector laplacian” depending upon whether it is operated with a scalar function or a vector, respectively.

- (a) Both A and R are true and R is a correct explanation of A.  
 (b) Both A and R are true but R is not a correct explanation of A.  
 (c) A is true but R is false.  
 (d) A is false but R is true.

**Q.26** Match **List-I (Terms)** with **List-II (Type)** and select the correct answer using the codes given below the lists:

**List-I**

- A.  $\text{Curl } (\vec{F}) = 0$   
 B.  $\text{Div } (\vec{F}) = 0$   
 C.  $\text{Div grad } (\phi) = 0$   
 D.  $\text{Div div } (\phi) = 0$

**List-II**

1. Laplace equation  
 2. Irrotational  
 3. Solenoidal  
 4. Not defined

**Codes:**

	A	B	C	D
(a)	2	3	1	4
(b)	4	1	3	2
(c)	2	1	3	4
(d)	4	3	1	2

**Q.27** Which of the following relations are not correct?

- (a)  $[B \times C, C \times A, A \times B] = [ABC]^2$   
 (b)  $A \times [B \times (C \times D)] = B \cdot D (A \times C) - B \cdot C (A \times D)$   
 (c)  $(B \times C) \cdot (A \times D) + (C \times A) \cdot (B \times D) + (A \times B) \cdot (C \times D) = 0$   
 (d)  $(A \times B)^2 = A^2 B^2 - (A \cdot B)^2$

**Q.28** If  $uF = \nabla v$ , where  $u$  and  $v$  are scalar fields and  $F$  is a vector field, then  $F \cdot \text{curl } F$  is equal to

- (a) zero  
 (b)  $\frac{\nabla^2 v}{u^2}$   
 (c)  $\frac{(\nabla v \cdot \nabla) v}{u^2}$   
 (d) not defined

**Q.29** Which of the following option is not correct?

- (a) A vector field  $\vec{A}$  is solenoid, if  $\nabla \cdot \vec{A} = 0$   
 (b) A vector field  $\vec{A}$  is irrotational, if  $\nabla \times \vec{A} = 0$   
 (c) A vector field  $V$  is harmonics, if  $\nabla^2 V \neq 0$   
 (d) All options are correct

**Q.30** Which of the following statements is not true of a phasor?

- (a) It may be a scalar or a vector.  
 (b) It is a time dependent quantity.  
 (c) It is a complex quantity.  
 (d) All are true

**Q.31** A scalar function,  $V$  is given by  $V = xyz^2$ . The gradient of  $V$  is given by

- (a)  $xz^2 \hat{a}_x + 2xyz \hat{a}_y + xz^2 \hat{a}_z$   
 (b)  $yz^2 \hat{a}_x + xz^2 \hat{a}_y + xyz \hat{a}_z$   
 (c)  $2xyz \hat{a}_z + yz^2 \hat{a}_y + xz^2 \hat{a}_z$   
 (d)  $yz^2 \hat{a}_x + xz^2 \hat{a}_y + 2xyz \hat{a}_z$

**Q.32** The scalar potential is given by

$$V = (x^2 - y^2 - z^2) \text{ Volts}$$

The laplacian of  $V$  is

- (a) 0  
 (b) -2  
 (c) 1  
 (d) -1

## 2. Electrostatics

**Q.33** What is the ratio of electrostatic repulsion and gravitational attraction between two electrons?  
 Given:

$$Q = \text{Charge on an electron} = 1.6 \times 10^{-19} \text{C}$$

$$G = \text{Gravitation constant} = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$m = \text{Mass of an electron} = 9.1 \times 10^{-31} \text{ kg}$$

- (a)  $3.23 \times 10^{42}$   
 (b)  $4.17 \times 10^{42}$   
 (c)  $2.33 \times 10^{42}$   
 (d)  $1.47 \times 10^{42}$

**Q.34** What is the electric field strength at a distance of 200 mm from a charge of  $2 \times 10^{-6}$  Coulomb in vacuum?

- (a) 450 kV/m  
 (b) 236 kV/m  
 (c) 525 kV/m  
 (d) 328 kV/m

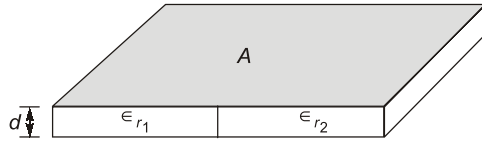
**Q.35** Two point charges of  $3 \times 10^{-9}$  C and  $-2 \times 10^{-9}$  C are spaced two meters apart. What is the electric field at a point which is one meter from each of the two point charges?

- (a) 11 V/m  
 (b) 5 V/m  
 (c) 9 V/m  
 (d) 6 V/m

**Q.36** Five equal point charges of  $Q = 20 \times 10^{-9}$  C, are placed at  $x = 2, 3, 4, 5$  and  $6$  cm. The electric potential at origin will be

- (a) 416 V  
 (b) 128 V  
 (c) 325 V  
 (d) 261 V

- Q.37** What is the equivalent capacitance of a parallel plate capacitor having two dielectrics,  $\epsilon_{r1} = 1.5$  and  $\epsilon_{r2} = 3.5$ , each comprising one-half the volume as shown in figure below with  $A = 2 \text{ m}^2$  and  $d = 1 \text{ mm}$ ?



- (a) 44.27 nF                      (b) 24.47 nF  
(c) 27.44 nF                      (d) 42.47 nF
- Q.38** Four point charges 1, -2, -3 and 4  $\mu\text{C}$  are located on the  $x$ -axis at  $x = 1, 2, 3$  and 4 metre respectively. The energy stored in the field is  
(a) -2 Joule                      (b) 4 Joule  
(c) 0 Joule                      (d) -1 Joule
- Q.39** The electric field strength at a point in front of an infinite sheet of charge is  
(a) independent of the distance of the point from the sheet  
(b) inversely proportional to the distance of the point from the sheet  
(c) inversely proportional to the square of distance of the point from the sheet  
(d) none of the above
- Q.40** **Assertion (A):** If all the points in space which have the same potential are jointed, then equipotential surfaces are obtained.  
**Reason (R):** The field and the equipotential surfaces are orthogonal to each other.  
(a) Both A and R are true and R is a correct explanation of A.  
(b) Both A and R are true but R is not a correct explanation of A.  
(c) A is true but R is false.  
(d) A is false but R is true.
- Q.41** Consider the following statements associated with various medias encountered in electrostatics:  
1. An isotropic media is one whose properties are independent of direction  
2. A homogeneous media is one whose physical characteristics vary from point-to-point.

3. A non-homogenous media is one whose physical characteristics do not vary from point-to-point.  
4. A linear media is a one in which the electric flux density is proportional to the electric field intensity.

Which of the above statements are correct?

- (a) 2 and 3                      (b) 3 and 4  
(c) 1, 2 and 4                      (d) 1 and 4

- Q.42** **Assertion (A):** Continuity of current equation is derived from law of conservation of charge.

**Reason (R):** Continuity of current equation states that there can be no accumulation of charge at any point.

- (a) Both A and R are true and R is a correct explanation of A.  
(b) Both A and R are true but R is not a correct explanation of A.  
(c) A is true but R is false.  
(d) A is false but R is true.

- Q.43** Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

**List-I**

**List-II**

- |                        |  |
|------------------------|--|
| A. Joule's law         | 1. $\vec{J} = \sigma \vec{E}$                                |
| B. Ohm's law           | 2. $\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t}$ |
| C. Solenoidal current  | 3. $W = I^2 R t$   |
| D. Continuity equation | 4. $\oint_s \vec{J} d\vec{s} = 0$                            |

**Codes:**

- |     | A | B | C | D |
|-----|---|---|---|---|
| (a) | 4 | 2 | 3 | 1 |
| (b) | 3 | 1 | 2 | 4 |
| (c) | 3 | 1 | 4 | 2 |
| (d) | 4 | 3 | 1 | 2 |

- Q.44** **Assertion (A):** An electric field produces no migration of charge in a dielectric.

**Reason (R):** The permittivity of a dielectric is always less than the permittivity of vacuum.

- (a) Both A and R are true and R is a correct explanation of A.  
(b) Both A and R are true but R is not a correct explanation of A.  
(c) A is true but R is false.  
(d) A is false but R is true.

**Q.45** Which one of the following pairs is NOT correctly matched?

- (a) Gauss theorem:  $\oint_S \vec{D} \cdot d\vec{s} = \oint_S \nabla \cdot \vec{D} dV$
- (b) Gauss's law:  $\oint \vec{D} \cdot d\vec{s} = \int \rho dV$
- (c) Coulomb's law:  $V = -\frac{d\phi_m}{dt}$
- (d) Stoke's theorem:  $\oint_L \vec{E} \cdot d\vec{l} = \iint_S (\nabla \times \vec{E}) \cdot d\vec{s}$

**Q.46** Match List-I (Law/Quantity) with List-II (Mathematical expression) and select the correct answer using the codes given below the lists:

**List-I**

- A. Gauss's law
- B. Ampere's law
- C. Faraday's law
- D. Poynting vector

**List-II**

1.  $\nabla \cdot \vec{D} = \rho$
2.  $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
3.  $s = \vec{E} \times \vec{H}$
4.  $\vec{F} = q(\vec{E} \times \vec{v}\vec{B})$
5.  $\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$

**Codes:**

	A	B	C	D
(a)	1	2	4	3
(b)	3	5	2	1
(c)	1	5	2	3
(d)	3	2	4	1

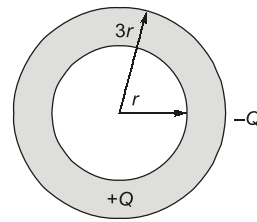
**Q.47** What is the value of total electric flux coming out of a closed surface?

- (a) Zero
- (b) Equal to volume charge density
- (c) Equal to the total charge enclosed by the surface
- (d) Equal to the surface charge density

**Q.48** Which one of the following is the Poisson's equation for a linear and isotropic but in homogeneous medium?

- (a)  $\nabla^2 V = -\rho/\epsilon$       (b)  $\vec{\nabla} \cdot (\epsilon \vec{\nabla} V) = -\rho$
- (c)  $\vec{\nabla} \cdot \vec{\nabla}(\epsilon V) = -\rho$       (d)  $\nabla^2 V = -\rho/\epsilon$

**Q.49** Two concentric spherical shells carry equal and opposite uniformly distributed charges over their surfaces as shown in figure below.



Electric field on the surface of inner shell will be

- (a) zero      (b)  $\frac{Q}{4\pi\epsilon_0 r^2}$
- (c)  $\frac{Q}{8\pi\epsilon_0 r^2}$       (d)  $\frac{Q}{16\pi\epsilon_0 R^2}$

**Q.50** Electric field intensity due to line charge of infinite length is

- (a)  $\frac{\rho_L}{2\pi\epsilon r}$       (b)  $\frac{\rho_L}{4\pi\epsilon r}$
- (c)  $\frac{\rho_L}{\pi\epsilon r}$       (d)  $\frac{2\rho_L}{\pi\epsilon r}$

**Q.51** Consider the following statements associated with equipotential surface:

- Potential is same everywhere.
- No current flows on this surface.
- Work done in moving charge from one point to another is zero.
- Potential is different everywhere.

Which of the above statements is/are not correct?

- (a) 1 and 3 only      (b) 3 and 4 only
- (c) 4 only      (d) 2 and 4 only

**Q.52** A finite sheet  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$  on the  $z = 0$  plane has a charge density

$$\rho_s = xy(x^2 + y^2 + 25)^{3/2} \text{ nC/m}^2$$

The total charge on the sheet would be

- (a)  $\left( \frac{(25)^{7/2} - (27)^{7/2} - 2(26)^{7/2}}{25} \right) \text{ nC}$
- (b)  $\left( \frac{(26)^{7/2} - 2(25)^{7/2} + (27)^{7/2}}{35} \right) \text{ nC}$
- (c)  $\left( \frac{(26)^{7/2} - (27)^{7/2} + 2(25)^{7/2}}{25} \right) \text{ nC}$
- (d)  $\left( \frac{(27)^{7/2} + (25)^{7/2} - 2(26)^{7/2}}{35} \right) \text{ nC}$

**Q.53 Assertion (A):** Gauss's law is an alternative statement of Coulomb's law.

**Reason (R):** Proper application of the divergence theorem to Coulomb's law results in Gauss's law.

- (a) Both A and R are true and R is a correct explanation of A.  
 (b) Both A and R are true but R is not a correct explanation of A.  
 (c) A is true but R is false.  
 (d) A is false but R is true.

**Q.54** If the potential  $V = \frac{10}{r^2} \sin\theta \cos\phi$ , then the work

done in moving a  $10 \mu\text{C}$  charge from point  $A(1, 30^\circ, 120^\circ)$  to  $B(4, 90^\circ, 60^\circ)$  would be equal to

- (a)  $16.65 \mu\text{J}$  (b)  $28.125 \mu\text{J}$   
 (c)  $25.25 \mu\text{J}$  (d)  $30.625 \mu\text{J}$

**Q.55** Three point charges  $-1 \text{ nC}$ ,  $4 \text{ nC}$  and  $3 \text{ nC}$  are located at  $(0, 0, 0)$ ,  $(0, 0, 1)$  and  $(1, 0, 0)$  respectively. To total energy contained in the system would be

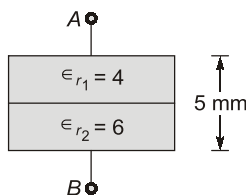
- (a)  $22.25 \text{ nJ}$  (b)  $15.56 \text{ nJ}$   
 (c)  $12.84 \text{ nJ}$  (d)  $13.37 \text{ nJ}$

**Q.56** A wire of diameter  $1 \text{ mm}$  and conductivity  $5 \times 10^7 \text{ S/m}$  has  $10^{29}$  free electrons  $/\text{m}^3$  when an electric field of  $10 \text{ mV/m}$  is applied.

The current density and the drift velocity of the electrons will be respectively given by (take charge on an electrons  $e = -1.6 \times 10^{-19} \text{ C}$ )

- (a)  $500 \text{ kA/m}^2$  and  $3.125 \times 10^{-5} \text{ m/s}$   
 (b)  $300 \text{ kA/m}^2$  and  $2.225 \times 10^{-5} \text{ m/s}$   
 (c)  $500 \text{ kA/m}^2$  and  $2.225 \times 10^{-5} \text{ m/s}$   
 (d)  $300 \text{ kA/m}^2$  and  $3.125 \times 10^{-5} \text{ m/s}$

**Q.57** The equivalent capacitance between the terminals  $A$  and  $B$  for the capacitor shown below would be (the area occupied by each dielectric is  $30 \text{ cm}^2$ )



- (a)  $26.53 \text{ pF}$  (b)  $28.68 \text{ pF}$   
 (c)  $25.46 \text{ pF}$  (d)  $22.22 \text{ pF}$

**Q.58** The cylindrical coordinates equation

$$\frac{\partial^2 \psi}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \psi}{\partial \rho} + \frac{\partial^2 \psi}{\partial z^2} + 10 = 0$$

- is called  
 (a) Laplace's equation  
 (b) Poisson's equation  
 (c) Larentz's equation  
 (d) Maxwell's equation

**Q.59** Image theory is applicable to problems involving

- (a) electrostatic field only  
 (b) magnetostatic fields only  
 (c) both electrostatic and magnetostatic fields  
 (d) neither electrostatic nor magnetostatic fields

**Q.60** The potential field at any point in space containing a dielectric material of relative permittivity 3 is given by

$V = (5x^2y + 3yz^2 + 6xz)$  Volts, where  $x, y, z$  are in meters. The magnetude of volume charge density at point  $(1, 2, 3)$  will be given by

- (a)  $48 \epsilon_0$  (b)  $96 \epsilon_0$   
 (c)  $144 \epsilon_0$  (d)  $192 \epsilon_0$

**Q.61** If  $\vec{A}$  and  $\vec{J}$  are the vector potential and current density vectors associated with a coil, then

$\int_V \vec{A} \cdot \vec{J} dV$  has the unit of

- (a) flux-linkage (b) power  
 (c) energy (d) inductance

**Q.62** In the infinite plane,  $y = 6 \text{ m}$ , there exists a uniform surface charge density of  $(1/600\pi) \mu\text{C/m}^2$ . The associated electric field strength is

- (a)  $30\hat{i} \text{ V/m}$  (b)  $30\hat{j} \text{ V/m}$   
 (c)  $30\hat{k} \text{ V/m}$  (d)  $60\hat{i} \text{ V/m}$

**Q.63** Consider the following statements:

1. Electric field intensity at any point is the negative of the potential gradient at that point.
2. Electric field due to a finite line of charge at a point is independent of the length of the line of charge.
3. The direction of the electric field is the direction in which the gradient is greatest.
4. Electric field due to an infinite line of charge at a point is inversely proportional to square of the distance of the point from the infinite line charge.

Answers Electromagnetic Theory							
1. (c)	2. (b)	3. (a)	4. (d)	5. (a)	6. (a)	7. (c)	8. (b)
9. (d)	10. (c)	11. (a)	12. (b)	13. (d)	14. (c)	15. (d)	16. (b)
17. (c)	18. (a)	19. (c)	20. (d)	21. (c)	22. (b)	23. (a)	24. (d)
25. (d)	26. (a)	27. (d)	28. (a)	29. (c)	30. (a)	31. (d)	32. (b)
33. (b)	34. (a)	35. (c)	36. (d)	37. (a)	38. (c)	39. (a)	40. (b)
41. (d)	42. (a)	43. (c)	44. (c)	45. (c)	46. (c)	47. (c)	48. (b)
49. (a)	50. (a)	51. (c)	52. (d)	53. (a)	54. (b)	55. (d)	56. (a)
57. (c)	58. (b)	59. (a)	60. (b)	61. (c)	62. (b)	63. (d)	64. (a)
65. (d)	66. (b)	67. (a)	68. (d)	69. (a)	70. (a)	71. (c)	72. (b)
73. (d)	74. (a)	75. (d)	76. (d)	77. (b)	78. (a)	79. (b)	80. (c)
81. (d)	82. (c)	83. (b)	84. (b)	85. (a)	86. (d)	87. (c)	88. (a)
89. (b)	90. (a)	91. (b)	92. (c)	93. (b)	94. (d)	95. (a)	96. (b)
97. (a)	98. (b)	99. (a)	100. (b)	101. (c)	102. (a)	103. (c)	104. (b)
105. (b)	106. (a)	107. (d)	108. (c)	109. (b)	110. (a)	111. (c)	112. (b)
113. (c)	114. (d)	115. (a)	116. (b)	117. (c)	118. (d)	119. (a)	120. (a)
121. (b)	122. (a)	123. (a)	124. (c)	125. (b)	126. (d)	127. (c)	128. (d)
129. (a)	130. (c)	131. (a)	132. (c)	133. (a)	134. (b)	135. (d)	136. (b)
137. (a)	138. (c)	139. (d)	140. (a)	141. (c)	142. (b)	143. (d)	144. (a)
145. (c)	146. (b)	147. (d)	148. (c)	149. (c)	150. (b)	151. (c)	152. (d)
153. (a)	154. (d)	155. (c)	156. (c)	157. (a)	158. (b)	159. (d)	160. (c)
161. (c)	162. (b)	163. (b)	164. (a)	165. (b)	166. (b)	167. (a)	168. (c)
169. (d)	170. (b)	171. (b)	172. (a)	173. (d)	174. (d)	175. (b)	176. (b)
177. (c)	178. (a)	179. (c)	180. (c)	181. (b)	182. (d)	183. (a)	184. (b)
185. (c)	186. (a)	187. (d)	188. (a)	189. (a)	190. (b)	191. (a)	192. (c)
193. (d)	194. (c)	195. (a)	196. (b)	197. (a)	198. (d)	199. (a)	200. (a)
201. (c)	202. (b)	203. (c)	204. (c)	205. (b)	206. (b)	207. (d)	

## Explanations

1. (c)

Here,

$$\begin{aligned}\text{Div. } \vec{Q} &= \nabla \cdot \vec{Q} = \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (\vec{i}zx) \\ &= (\vec{i} \cdot \vec{i}) \left( \frac{\partial}{\partial x} \cdot zx \right) = 1 \cdot z = z\end{aligned}$$

Also,

$$\text{Grad } \phi = \nabla \phi = \left( \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right) (xy^2z^3) = (y^2z^3 \vec{i} + 2yxz^3 \vec{j} + 3z^2y^2x \vec{k})$$

And,

$$\begin{aligned}\text{Curl } \vec{P} = \nabla \times \vec{P} &= \begin{vmatrix} \vec{i} & 0 & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & 0 & (x-y) \end{vmatrix} \\ &= \vec{i} \left[ \frac{\partial}{\partial y} (x-y) - 0 \right] + \vec{k} \left[ 0 - \frac{\partial}{\partial y} (x^2y) \right] \\ &= \vec{i} [-1] + \vec{k} [x^2] = -\vec{i} + x^2 \vec{k}\end{aligned}$$

2. (b)

Let the unit vector be given by  $\vec{a}_R$ .

Now,

$$\begin{aligned}\vec{R} &= \text{Difference of two vectors} \\ &= r\vec{a}_r - h\vec{a}_z\end{aligned}$$

$\therefore$  Unit vector,

$$\vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$$

3. (a)

Since the given vector  $V$  is irrotational, therefore  $\text{curl } V = 0$  or,  $\nabla \times V = 0$ .

Now,

$$\begin{aligned}\nabla \times V &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x+2y+az) & (bx-3y-z) & (4x+cy+2z) \end{vmatrix} \\ &= \left\{ \frac{\partial}{\partial y} (4x+cy+2z) - \frac{\partial}{\partial z} (bx-3y-z) \right\} \vec{i} + \left\{ \frac{\partial}{\partial z} (x+2y+az) - \frac{\partial}{\partial x} (4x+cy+2z) \right\} \vec{j} \\ &\quad + \left\{ \frac{\partial}{\partial x} (bx-3y-z) - \frac{\partial}{\partial y} (x+2y+az) \right\} \vec{k} \\ &= (c+1)\vec{i} + (a-4)\vec{j} + (b-2)\vec{k}\end{aligned}$$

Since,  $\nabla \times V = 0$ , therefore,  $a = 4$ ,  $b = 2$ , and  $c = -1$

4. (d)

- $(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D})$  is called "product of four vectors".

- $\vec{A} \times (\vec{B} \times \vec{C})$  is called "vector triple product".
- $(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D})$  is called "vector product of four vectors".

**6. (a)**

Both assertion and reason are true and reason is the correct explanation of assertion. Reason is the physical interpretation of divergence.

**7. (c)**

- $\vec{A} \times (\vec{B} \times \vec{C})$  is called "vector triple product" which is a correct expression.

$$\nabla = \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right)$$

and let  $\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$

$$\text{Then, } \nabla \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

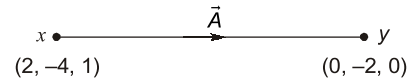
$$\begin{aligned} \therefore \nabla(\nabla \times \vec{A}) &= \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \times \left[ \vec{i} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \vec{j} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \vec{k} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \right] \\ &= \frac{\partial}{\partial x} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \frac{\partial}{\partial y} \left( \frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) + \frac{\partial}{\partial z} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\ &= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} + \frac{\partial^2 A_z}{\partial y \partial x} - \frac{\partial^2 A_x}{\partial y \partial z} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y} = 0 \end{aligned}$$

$$\begin{aligned} \nabla \times \nabla \phi &= \left( \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right) \times \left( \frac{\partial \phi}{\partial x} \vec{i} + \frac{\partial \phi}{\partial y} \vec{j} + \frac{\partial \phi}{\partial z} \vec{k} \right) \\ &= \left\{ (\vec{i} \times \vec{j}) \frac{\partial^2 \phi}{\partial x \partial y} + (\vec{i} \times \vec{k}) \frac{\partial^2 \phi}{\partial x \partial z} \right\} + \left\{ (\vec{j} \times \vec{i}) \frac{\partial^2 \phi}{\partial x \partial y} + (\vec{j} \times \vec{k}) \frac{\partial^2 \phi}{\partial y \partial z} \right\} + \left\{ (\vec{k} \times \vec{i}) \frac{\partial^2 \phi}{\partial z \partial x} + (\vec{k} \times \vec{j}) \frac{\partial^2 \phi}{\partial y \partial z} \right\} \\ &= \vec{k} \frac{\partial^2 \phi}{\partial x \partial y} - \vec{j} \frac{\partial^2 \phi}{\partial x \partial z} - \vec{k} \frac{\partial^2 \phi}{\partial y \partial x} + \vec{i} \frac{\partial^2 \phi}{\partial y \partial z} + \vec{j} \frac{\partial^2 \phi}{\partial z \partial x} - \vec{i} \frac{\partial^2 \phi}{\partial z \partial y} = 0 \end{aligned}$$

**8. (b)**

The vector  $\vec{A}$  is given as

$$\begin{aligned} \vec{A} &= (0 - 2)\vec{a}_x + [-2 - (-4)]\vec{a}_y + (0 - 1)\vec{a}_z \\ &= -2\vec{a}_x + 2\vec{a}_y - \vec{a}_z \end{aligned}$$

**9. (d)**

By divergence theorem,

$$\begin{aligned} \iiint_s \vec{F} \cdot d\vec{s} &= \iiint_v \nabla \cdot \vec{F} dv \\ &= \iiint_v \left[ \vec{i}_1 \frac{\partial}{\partial x} + \vec{i}_2 \frac{\partial}{\partial y} + \vec{i}_3 \frac{\partial}{\partial z} \right] \times (4xz\vec{i}_1 - y^2\vec{i}_2 + yz\vec{i}_3) dv \end{aligned}$$

$$\begin{aligned}
 &= \iiint_V \left[ \frac{\partial}{\partial x}(4xz) - \frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial z}(yz) \right] dV \\
 &= \iiint_V [4z - 2y + y] dV = \iiint_V [4z - y] dV
 \end{aligned}$$

Since, the surface  $s$  is bounded by  $x = 0, 1$ ;  $y = 0, 1$  and  $z = 0, 1$  so, putting the limits, we have:

$$\begin{aligned}
 \iint_s \vec{F} \cdot d\vec{s} &= \iiint_{000}^{111} (4z - y) dx dy dz = \int_0^1 \int_0^1 \left( \frac{4z^2}{2} - yz \right)_0^1 dx dy \\
 &= \int_0^1 \int_0^1 (2 - y) dx dy = \int_0^1 \left( 2y - \frac{y^2}{2} \right)_0^1 dy = \int_0^1 \frac{3}{2} dy = \frac{3}{2}
 \end{aligned}$$

### 10. (c)

The vector field  $\vec{A}$  will be irrotational, if  $\nabla \times \vec{A} = 0$ .

Now,

$$\begin{aligned}
 \nabla \times \vec{A} &= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} \\
 &= \left[ \frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(xz) \right] \vec{a}_x + \left[ \frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz) \right] \vec{a}_y + \left[ \frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(yz) \right] \vec{a}_z \\
 &= [x - x] \vec{a}_x + [y - y] \vec{a}_y + [z - z] \vec{a}_z = 0
 \end{aligned}$$

Hence,  $\vec{A}$  is irrotational.

The vector field  $\vec{A}$  will be solenoidal, if  $\nabla \cdot \vec{A} = 0$

Here,

$$\begin{aligned}
 \nabla \cdot \vec{A} &= \left( \vec{a}_x \frac{\partial}{\partial x} + \vec{a}_y \frac{\partial}{\partial y} + \vec{a}_z \frac{\partial}{\partial z} \right) \cdot (yz \vec{a}_x + xz \vec{a}_y + xy \vec{a}_z) \\
 &= \vec{a}_x \cdot \vec{a}_x \frac{\partial}{\partial x}(yz) + \vec{a}_y \cdot \vec{a}_y \frac{\partial}{\partial y}(xz) + \vec{a}_z \cdot \vec{a}_z \frac{\partial}{\partial z}(xy) = 0 + 0 + 0 = 0
 \end{aligned}$$

Hence,  $\vec{A}$  is solenoidal.

### 11. (a)

Given,

$$\vec{A} = \frac{1}{\sqrt{x^2 + y^2}} \vec{a}_x$$

$\therefore$

$$\begin{aligned}
 \nabla \cdot \vec{A} &= \frac{\partial}{\partial x}(A_x) + \frac{\partial}{\partial y}(A_y) + \frac{\partial}{\partial z}(A_z) \\
 &= \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{x^2 + y^2}} \right) + 0 + 0 = \frac{\partial}{\partial x} (x^2 + y^2)^{-1/2} = -\frac{1}{2} (x^2 + y^2)^{-3/2} \cdot 2x \\
 &= \nabla \cdot \vec{A} = -\frac{x}{\sqrt{(x^2 + y^2)}(x^2 + y^2)}
 \end{aligned}$$

Now,

$$(\nabla \cdot \vec{A})_{2,2,0} = -\frac{2}{\sqrt{(2^2 + 2^2)} \cdot (2^2 + 2^2)} = -\frac{2}{\sqrt{8} \cdot 8} = -0.0884$$



**12. (b)**

Given,  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$   
 $\therefore \vec{r} \times \vec{i} = (x\vec{i} + y\vec{j} + z\vec{k}) \times \vec{i} = -y\vec{k} + z\vec{j}$   
 Also,  $\vec{i} \times (\vec{r} \times \vec{i}) = \vec{i} \times (-y\vec{k} + z\vec{j}) = \vec{j}y + z\vec{k}$   
 Similarly,  $\vec{j} \times (\vec{r} \times \vec{j}) = \vec{i}x + \vec{k}z$   
 and  $\vec{k} \times (\vec{r} \times \vec{k}) = \vec{i}x + \vec{j}y$   
 Thus,  $\vec{i} \times (\vec{r} \times \vec{i}) + \vec{j} \times (\vec{r} \times \vec{j}) + \vec{k} \times (\vec{r} \times \vec{k}) = 2(x\vec{i} + y\vec{j} + z\vec{k}) = 2\vec{r}$

**13. (d)**

Since vector  $\vec{V}$  is solenoidal, therefore

$$\nabla \cdot \vec{V} = 0$$

$$\therefore \left[ \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right] \cdot \begin{bmatrix} \vec{i}(x+3y) + \vec{j}(y-2x) \\ + \vec{k}(x+bz) \end{bmatrix} = 0$$

$$\text{or, } [1 + 1 + b] = 0 \text{ or } b = -2$$

**15. (d)**

Vector differential operator (' $\nabla$ ') is not a vector quantity. Hence, assertion is a false statement.

**16. (b)**

All the given statements are correct.

**17. (c)**

Reason is a statement of stroke's theorem not that of Gauss's divergence theorem.

**18. (a)**

- Electric potential,  $V = \frac{\text{Work done}}{\text{Test charge}} = \frac{\text{Force} \times \text{Displacement}}{\text{Current} \times \text{Time}}$

$$\therefore [V] = \frac{[MLT^{-2}][L]}{[I][T]} = [ML^2T^{-3}I^{-1}]$$

- Emf induced,  $V = \frac{d\phi}{dt}$   
 or,  $\phi = V \cdot t = \text{Magnetic flux}$   
 $\therefore [\phi] = [ML^2T^{-3}I^{-1}][T] = [ML^2T^{-2}I^{-1}]$
- Magnetic field intensity,

$$H = \frac{NI}{l}$$

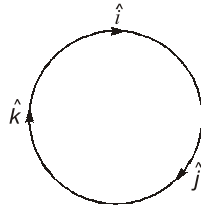
$$\therefore [H] = \frac{[I]}{[L]} = [IL^{-1}]$$

- Magnetic flux density  $B = \frac{\phi}{A}$

$$\therefore [B] = \frac{[ML^2T^{-2}I^{-1}]}{[L^2]} = [MT^{-2}I^{-1}]$$

**19. (c)**

Option (c) is not correct because cross product of two unlike vectors is a third unit vector having positive sign for normal rotation and negative for reverse rotation while cross product of two like unit vectors is zero.

**20. (d)**

Taking the curl, we have:

$$\nabla \times \vec{v} = \nabla \times \vec{\omega} \times \vec{r} \quad (\text{Since } \vec{v} = \vec{\omega} \times \vec{r})$$

$$\begin{aligned} \text{or, } \nabla \times \vec{v} &= \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \times \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix} \\ &= \left[ \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right] \times \left[ \vec{i}(\omega_y z - \omega_z y) + \vec{j}(\omega_z x - \omega_x z) + \vec{k}(\omega_x y - \omega_y x) \right] \\ &= (\vec{i} \times \vec{j}) \frac{\partial}{\partial x} (\omega_z x - \omega_x z) + (\vec{i} \times \vec{k}) \frac{\partial}{\partial x} (\omega_x y - \omega_y x) + (\vec{j} \times \vec{i}) \frac{\partial}{\partial y} (\omega_y z - \omega_z y) \\ &\quad + (\vec{j} \times \vec{k}) \frac{\partial}{\partial y} (\omega_x y - \omega_y x) + (\vec{k} \times \vec{i}) \frac{\partial}{\partial z} (\omega_y z - \omega_z y) + (\vec{k} \times \vec{j}) \frac{\partial}{\partial z} (\omega_z x - \omega_x z) \\ &= \vec{k}(\omega_z - 0) - \vec{j}(0 - \omega_y) - \vec{k}(0 - \omega_z) + \vec{i}(\omega_x - 0) + \vec{j}(\omega_y - 0) - \vec{i}(0 - \omega_x) \\ &= \vec{i}(\omega_x + \omega_x) + \vec{j}(\omega_y + \omega_y) + \vec{k}(\omega_z + \omega_z) \\ &= 2\omega_x \vec{i} + 2\omega_y \vec{j} + 2\omega_z \vec{k} = 2(\omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}) = 2\vec{\omega} \end{aligned}$$

$$\therefore \nabla \times \vec{v} = \text{Curl } \vec{v} = 2\vec{\omega}$$

**21. (c)**

We have:

$$\begin{aligned} \nabla \vec{r} &= \left( \vec{i}_x \frac{\partial}{\partial x} + \vec{i}_y \frac{\partial}{\partial y} + \vec{i}_z \frac{\partial}{\partial z} \right) (x\vec{i}_x + y\vec{i}_y + z\vec{i}_z) \\ &= (\vec{i}_x \cdot \vec{i}_x) \frac{\partial x}{\partial x} + (\vec{i}_y \cdot \vec{i}_y) \frac{\partial y}{\partial y} + (\vec{i}_z \cdot \vec{i}_z) \frac{\partial z}{\partial z} \\ &= 1.1 + 1.1 + 1.1 = 1 + 1 + 1 = 3 \end{aligned}$$

Also,

$$\begin{aligned} \nabla \times \vec{r} &= \begin{vmatrix} \vec{i}_x & \vec{i}_y & \vec{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = \vec{i}_x \left[ \frac{\partial z}{\partial y} - \frac{\partial y}{\partial z} \right] + \vec{i}_y \left[ \frac{\partial x}{\partial z} - \frac{\partial z}{\partial x} \right] + \vec{i}_z \left[ \frac{\partial y}{\partial x} - \frac{\partial x}{\partial y} \right] \\ &= \vec{i}_x [0] + \vec{i}_y [0] + \vec{i}_z [0] = 0 + 0 + 0 = 0 \end{aligned}$$

Thus,

$$\nabla \times \vec{r} = 0$$

Hence, both (a) and (b) will hold true.

**22. (b)**

Let,  $\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$  be any vector field in cartesian co-ordinate system then, we can prove that

$$\nabla \times (\nabla \times \vec{E}) = \nabla \cdot (\nabla \times \vec{E}) - \nabla^2 \vec{E} \quad \text{or} \quad \nabla \cdot (\nabla \times \vec{E}) = \nabla \times \nabla \times \vec{E} + \nabla^2 \vec{E}$$

$$\begin{aligned} \text{Also, Div. Curl } \vec{E} = \nabla \cdot \nabla \times \vec{E} &= \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \\ &= \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot \left[ \vec{i} \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + \vec{j} \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + \vec{k} \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \right] \\ &= \frac{\partial}{\partial x} \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + \frac{\partial}{\partial y} \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = 0 \end{aligned}$$

**23. (a)**

Given,

$$\vec{b} = \vec{a} \times \vec{c} \quad \text{and} \quad \vec{c} = \vec{a} \times \vec{b}$$

$\therefore$

$$\vec{b} = \vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = a^2 \vec{b} - a^2 \vec{b} = 0$$

Also,

$$\vec{c} = \vec{a} \times \vec{b} = \vec{a} \times (\vec{a} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{a})\vec{c} = a^2 \vec{c} - a^2 \vec{c} = 0$$

**24. (d)**

$$\begin{aligned} \iint_s \vec{A} \cdot \vec{n} d\vec{s} &= \iiint_s \nabla \cdot \vec{A} dV \\ &= \iiint_v \left[ \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (\vec{i}ax + \vec{j}by + \vec{k}cz) \right] dV \\ &= \iiint_v \left[ \frac{\partial}{\partial x}(ax) + \frac{\partial}{\partial y}(by) + \frac{\partial}{\partial z}(cz) \right] dV = \iiint_v [(a+b+c)] dV \\ &= (a+b+c) \iiint_v dV = (a+b+c)V \end{aligned}$$

**25. (d)**

Assertion is not true because the laplacian operator ( $\nabla^2$ ) of a scalar function  $\phi$  can be defined as "Divergence of the gradient of the scalar  $\phi$ ". i.e.  $\nabla \cdot \nabla \phi$ .

**27. (d)**

$$(A \times B)^2 = A^2 B^2 - (A \cdot B)^2$$

**28. (a)**

Given,

$$F = \frac{1}{u} \nabla v$$

$\therefore$

$$\text{Curl } F = \nabla \times \left( \frac{1}{u} \nabla v \right)$$

or,

$$\text{Curl } F = \nabla \frac{1}{u} \times \nabla v + \frac{1}{u} \nabla \times (\nabla v) = \nabla \frac{1}{u} \times \nabla v$$

Hence,

$$F \cdot \text{Curl } F = \frac{1}{u} \nabla v \cdot \left( \nabla \frac{1}{u} \times \nabla v \right) = 0$$

**29. (c)**

A scalar field  $V$  is harmonic, if  $\nabla^2 V = 0$ . Hence, option (c) is not correct.

**30. (a)**

A phasor is always a vector quantity.

**31. (d)**

$$\text{Grad } V = \nabla V$$

$$\begin{aligned} &= \left( \frac{\partial V}{\partial x} \right) \hat{a}_x + \left( \frac{\partial V}{\partial y} \right) \hat{a}_y + \left( \frac{\partial V}{\partial z} \right) \hat{a}_z \\ &= \frac{\partial}{\partial x} (xyz^2) \hat{a}_x + \frac{\partial}{\partial y} (xyz^2) \hat{a}_y \\ &\quad + \frac{\partial}{\partial z} (xyz^2) \hat{a}_z \end{aligned}$$

$$\text{or, } \nabla V = yz^2 \hat{a}_x + xz^2 \hat{a}_y + 2xyz \hat{a}_z$$

**32. (b)**

$$\begin{aligned} \text{The laplacian of } V &= \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \\ &= 2 - 2 - 2 = -2 \end{aligned}$$

**33. (b)**

By coulomb's law,

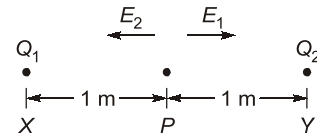
$$\begin{aligned} \text{electrostatic repulsion} &= \frac{1}{4\pi\epsilon_0} \times \frac{Q_1 Q_2}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \times \frac{Q^2}{r^2} = 9 \times 10^9 \times \frac{Q^2}{r^2} \\ &\quad \left( \because \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \right) \end{aligned}$$

Also, by Newton's law of gravitation,

$$\begin{aligned} \text{gravitational attraction} &= G \frac{m_1 m_2}{r^2} = G \frac{m^2}{r^2} \\ \therefore \frac{\text{Electrostatic repulsion}}{\text{Gravitational attraction}} &= \frac{9 \times 10^9 \times Q^2}{r^2} \times \frac{r^2}{Gm^2} \\ &= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times (9.1 \times 10^{-31})^2} \\ &= 4.17 \times 10^{42} \end{aligned}$$

**34. (a)**

$$\begin{aligned} E &= \frac{Q}{4\pi\epsilon_0 r^2} \text{ V/m} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.2)^2} \\ &= \frac{18}{0.04} \times 10^3 \text{ V/m} = 450 \text{ kV/m} \end{aligned}$$

**35. (c)**

Given,  $Q_1 = +3 \times 10^{-9} \text{ C}$  and  $Q_2 = -2 \times 10^{-9} \text{ C}$

$$\begin{aligned} \text{Here, } E_1 &= \frac{Q_1}{4\pi\epsilon_0 r^2} = \frac{9 \times 10^9 \times 3 \times 10^{-9}}{1^2} \\ &= 27 \text{ V/m} \quad (\text{Along } PY) \end{aligned}$$

$$\begin{aligned} \text{and } E_2 &= \frac{Q_2}{4\pi\epsilon_0 r^2} = \frac{9 \times 10^9 \times (-2 \times 10^{-9})}{1^2} \\ &= -18 \text{ V/m} \quad (\text{Along } PX) \end{aligned}$$

Hence, electric field intensity at point  $P$  is

$$E = E_1 + E_2 = 27 - 18 = 9 \text{ V/m}$$

**36. (d)**

Electric potential at origin is given as

$$V = \frac{1}{4\pi\epsilon_0} \left[ \frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \frac{Q_4}{r_4} + \frac{Q_5}{r_5} \right]$$

$$\begin{aligned} \text{Since, } Q_1 &= Q_2 = \dots Q_5 \\ &= Q = 20 \times 10^{-9} \text{ C} \end{aligned}$$

Therefore,

$$\begin{aligned} V &= 9 \times 10^9 \times 20 \times 10^{-9} \left[ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} \right] \\ &= 180 \left[ \frac{30 + 20 + 15 + 12 + 10}{60} \right] \text{ Volts} \\ &= 180 \times \frac{87}{60} \text{ V} = 3 \times 87 = 261 \text{ Volts} \end{aligned}$$

**37. (a)**

Given,  $\epsilon_{r1} = 1.5$ ,  $\epsilon_{r2} = 3.5$ ,

$$A_1 = A_2 = \frac{2}{2} \text{ m}^2 = 1 \text{ m}^2$$

and  $d = 1 \times 10^{-3} \text{ m}$

$$\begin{aligned} \text{Here, } C_1 &= \frac{\epsilon_0 \epsilon_{r1} A_1}{d} \\ &= \frac{8.854 \times 10^{-12} \times 1.5 \times (2/2)}{10^{-3}} \\ &= 13.281 \times 10^{-9} \text{ F} = 13.281 \text{ nF} \\ \text{and } C_2 &= \frac{\epsilon_0 \epsilon_{r2} A_2}{d} = \frac{8.854 \times 3.5 \times 10^{-12} \times 1}{10^{-3}} \\ &= 30.989 \text{ nF} \end{aligned}$$

$$\begin{aligned}\therefore C_{\text{equ}} &= C_1 + C_2 \\ &= 13.281 + 30.989 = 44.27 \text{ nF}\end{aligned}$$

**38. (c)**

Energy stored  $W = \frac{1}{2} VQ$  Joules

$$\begin{aligned}\therefore W &= \frac{1}{2} Q \left[ \frac{1}{4\pi\epsilon} \left\{ \frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \frac{Q_4}{r_4} \right\} \right] \\ &= \frac{Q}{8\pi\epsilon} \left[ \frac{1}{1} + \frac{-2}{2} + \frac{-3}{3} + \frac{4}{4} \right] \\ &= \frac{Q}{8\pi\epsilon} [1 - 1 - 1 + 1] = 0 \text{ Joule}\end{aligned}$$

**39. (a)**

The electric field strength at a point in front of an infinite sheet of charge is given by

$$\vec{E} = \frac{\rho_s}{\epsilon_0} \vec{a}_x \text{ Volts/meter}$$

where,  $\rho_s$  = charge density and

$\vec{a}_x$  = unit vector normal to the sheet and directed away from the sheet.

Here,  $\vec{E}$  is independent of the distance of the point from the sheet.

**40. (b)**

Both assertion and reason are individually true. However, the correct reason for assertion is that equipotential surfaces are those on which the potential is everywhere the same. Thus, if all the points in space which have the same potential are joined, then equipotential surfaces are obtained.

**41. (d)**

Statements 2 and 3 are not correct because a homogenous media is one whose physical characteristics do not vary from point-to-point while a non-homogeneous media is one whose physical characteristics vary from point-to-point.

**44. (c)**

- Assertion is a true statement.
- Reason is a false statement because the permittivity of dielectric is always more than the permittivity of vacuum which makes it a good insulator.

- In a dielectric or insulator the electrons are so tightly bound by their parent nuclei in the equilibrium positions that they can not easily be detached by the applications of electric field. Due to this reason an electric field produces no migration of charge in a dielectric.

**45. (c)**

Option (c) is Faraday's law given by  $V = -\frac{d\phi_m}{dt}$

**47. (c)**

From Gauss's law, total electric flux from a closed surface is  $\Psi = \oint \vec{D} \cdot d\vec{s} = Q_{\text{enclosed}}$

**48. (b)**

For homogeneous medium,  $\nabla^2 V = -\rho/\epsilon$  and for non-homogeneous medium  $\epsilon$  is a variable quantity due to which Poisson's equation becomes  $\vec{\nabla} \cdot (\epsilon \vec{\nabla} V) = -\rho$ .

**51. (c)**

Potential is same everywhere on an equipotential surface. Hence, statement 4 is not correct. Statements 1, 2 and 3 are correct.

**52. (d)**

The total charge on the sheet is

$$Q = \int \rho_s dS = \int_0^1 \int_0^1 xy(x^2 + y^2 + 25)^{3/2} dx dy \text{ nC}$$

Since,  $x dx = \frac{1}{2} d(x^2)$ , we now integrate w.r.t.  $x^2$

(or change variables:  $x^2 = u$  so that  $x dx = \frac{du}{2}$ )

$$\begin{aligned}\therefore Q &= \frac{1}{2} \int_0^1 y \int_0^1 (x^2 + y^2 + 25)^{3/2} d(x^2) dy \text{ nC} \\ &= \frac{1}{2} \int_0^1 y \cdot \frac{2}{5} (x^2 + y^2 + 25)^{5/2} \Big|_0^1 dy \\ &= \frac{1}{5} \int_0^1 \frac{1}{2} [(y^2 + 26)^{5/2} - (y^2 + 25)^{5/2}] d(y^2) \\ &= \frac{1}{10} \times \frac{2}{7} [(y^2 + 26)^{7/2} - (y^2 + 25)^{7/2}] \Big|_0^1 \\ &= \frac{1}{35} [(27)^{7/2} + (25)^{7/2} - 2(26)^{7/2}] \text{ nC}\end{aligned}$$

**54. (b)**

Required work done,

$$\begin{aligned}
 W &= -Q \int_A^B E \cdot dl = QV_{AB} \\
 &= Q(V_B - V_A) \\
 &= 10 \left( \frac{10}{16} \sin 90^\circ \cos 60^\circ - \frac{10}{1} \sin 30^\circ \cos 120^\circ \right) \times 10^{-6} \\
 &= 10 \left( \frac{10}{32} - \frac{-5}{2} \right) \times 10^{-6} = 28.125 \mu\text{J}
 \end{aligned}$$

**55. (d)**

Total energy contained in the system is:

$$\begin{aligned}
 W &= W_1 + W_2 + W_3 \\
 &= 0 + Q_2 V_{21} + Q_3 (V_{31} + V_{32}) \\
 &= Q_2 \cdot \frac{Q_1}{4\pi\epsilon_0 [(0,0,1) - (0,0,0)]} + \\
 &\quad \frac{Q_3}{4\pi\epsilon_0} \left[ \frac{Q_1}{[(1,0,0) - (0,0,0)]} + \frac{Q_2}{[(1,0,0) - (0,0,1)]} \right] \\
 &= \frac{1}{4\pi\epsilon_0} \left( Q_1 Q_2 + Q_1 Q_3 + \frac{Q_2 Q_3}{\sqrt{2}} \right) \\
 &= \frac{1}{4\pi \times \frac{10^{-9}}{36\pi}} \left( -4 - 3 + \frac{12}{\sqrt{2}} \right) \times 10^{-18} \\
 &= 9 \left( \frac{12}{\sqrt{2}} - 7 \right) \text{ nJ} = 13.37 \text{ nJ}
 \end{aligned}$$

**56. (a)**

The charge density of free electron is:

$$\begin{aligned}
 \rho_v &= ne = (10^{29}) \times (-1.6 \times 10^{-19}) \\
 &= -1.6 \times 10^{10} \text{ C/m}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Given, } E &= 10 \text{ mV/m,} \\
 d &= 1 \text{ mm,} \\
 \sigma &= 5 \times 10^7 \text{ S/m,} \\
 n &= 10^{29} / \text{m}^3
 \end{aligned}$$

Now, current density,

$$\begin{aligned}
 J &= \sigma E \\
 &= (5 \times 10^7) \times (10 \times 10^{-3}) \\
 &= 500 \text{ kA/m}^2
 \end{aligned}$$

Also, drift velocity,

$$\begin{aligned}
 V_d &= \frac{J}{\rho_v} = \frac{\text{Current density}}{\text{Charge density}} \\
 &= \frac{5 \times 10^5}{1.6 \times 10^{10}} = 3.125 \times 10^{-5} \text{ m/s}
 \end{aligned}$$

**57. (c)**

$$\text{Given, } d = 5 \text{ mm, } \frac{d}{2} = 2.5 \text{ mm,}$$

$$\epsilon_{r1} = 4, \epsilon_{r2} = 6,$$

$$\frac{A}{2} = 15 \text{ cm}^2$$

For the given arrangement of dielectrics,  $D$  and  $E$  are parallel to the dielectric interface. Therefore, we can treat the capacitor as consisting of two capacitors  $C_1$  and  $C_2$  in parallel (same voltage across  $C_1$  and  $C_2$ ).

$$\text{Thus, } C_1 = \frac{\epsilon_0 \epsilon_{r1} A}{d/2} = \frac{2 \epsilon_0 \epsilon_{r1} A}{d}$$

$$\text{and } C_2 = \frac{2 \epsilon_0 \epsilon_{r2} A}{d}$$

The total capacitance/equivalent capacitance between terminals  $A$  and  $B$  is

$$\begin{aligned}
 C_{AB} &= \frac{C_1 C_2}{C_1 + C_2} = \frac{2 \epsilon_0 A}{d} \left( \frac{\epsilon_{r1} \epsilon_{r2}}{\epsilon_{r1} + \epsilon_{r2}} \right) \\
 &= 2 \times \frac{10^{-9}}{36\pi} \times \frac{30 \times 10^{-4}}{5 \times 10^{-3}} \times \frac{4 \times 6}{10} \\
 &= 25.46 \text{ pF}
 \end{aligned}$$

**60. (b)**

Given,  $V = (5x^2y + 3yz^2 + 6xz)$  Volts and  $\epsilon_r = 3$   
Using Poisson's equation,

$$\nabla^2 V = -\frac{\rho_v}{\epsilon} \text{ or } \rho_v = -\epsilon \nabla^2 V$$

$$\begin{aligned}
 \text{Here, } \nabla^2 V &= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \\
 &= 10y + 0 + 6y = 16y
 \end{aligned}$$

$$\begin{aligned}
 \text{So, } \rho_v &= -\epsilon (10y + 6y) \\
 \therefore \rho_v|_{\text{at } (1,2,3)} &= -\epsilon_0 \epsilon_r (10 \times 2 + 6 \times 2) \\
 &= -\epsilon_0 \times 3 \times 32 = -96\epsilon_0 \\
 &= 96\epsilon_0 \text{ (Magnitude only)}
 \end{aligned}$$

**61. (c)**

On comparing  $W_m = \int_V \vec{A} \cdot \vec{J} dV$  with electrical

energy stored i.e.  $W = \frac{1}{2} \int_V \rho_v \cdot V dV$  we can find

that  $W_m$  has the unit of energy.

**62. (b)**Given,  $y = 6 \text{ m}$ 

$$\rho = \frac{1}{600\pi} \mu\text{C/m}^2$$

The electric field strength due to an infinite charged sheet is given by

$$\vec{E} = \left( \frac{\rho_s}{2\epsilon_0} \right) \hat{a}_n$$

$$\begin{aligned} \therefore \vec{E} &= \left( \frac{10^{-6}}{600\pi \times 2 \times 8.854 \times 10^{-12}} \right) \hat{a}_y \\ &= 30 \hat{a}_y \text{ V/m} = 30 \hat{j} \text{ V/m} \end{aligned}$$

**63. (d)**

- Since,  $\nabla V \cdot d\vec{s} = -E d\vec{s}$

$$\text{i.e. } \vec{E} = -\nabla V = -\text{Grad } V,$$

therefore electric field intensity at any point is the negative of the potential gradient at that point. Hence, statement-1 is correct.

- Electric field at a point due to a finite line of charge is given by

$$E = \frac{\rho_L a}{2\pi \epsilon_0 r \sqrt{a^2 + r^2}}$$

Where,

$2a$  = Length of finite lines of charge and

$r$  = Distance of the point on the  $r$ -axis from the finite line of charge.

Thus  $E$  depends on length of finite lines of charge.

Hence, statement-2 is not correct.

- Statement-3 is correct since  $\vec{E} = -\text{Grad } V$ .
- Electric field at a point due to an infinite line of charge is given by

$$E = \frac{\rho_L}{2\pi \epsilon_0 r}$$

(Where,  $r$  = Distance of the point from the infinite line of the charge)

Thus,  $E \propto \frac{1}{r}$ . Hence, statement-4 is not correct.

**64. (a)**

$$\begin{aligned} \vec{F} &= \frac{Q_1 Q_2}{4\pi \epsilon_0 r^2} \vec{a}_z \text{ N} \\ &= \frac{2 \times 10^{-6} \times 10 \times 10^{-6}}{4\pi \epsilon_0 \times (0.3)^2} \vec{a}_z \text{ N} \end{aligned}$$

$$= \frac{20 \times 10^{-12} \times 9 \times 10^9}{0.09} \vec{a}_z \text{ N}$$

$$= \frac{20 \times 10^{-12} \times 10^{11} \times 9}{9} \vec{a}_z = 2 \vec{a}_z \text{ N}$$

$$\therefore |\vec{F}| = 2 \text{ Newton}$$

**65. (d)**

Since  $\vec{D}$  is constant over the area and perpendicular to it, therefore flux  $\psi = D \cdot A$  Coulomb.

Given,  $A = 1 \text{ m}^2$ ,

$$\vec{D} = 10x \vec{a}_x$$

$$\therefore \left| \vec{D} \right|_{x=3} = 10 \times 3 \vec{a}_x = 30 \vec{a}_x$$

$$\text{or, } \left| \vec{D} \right| = 30 \text{ C/m}^2$$

$$\therefore \psi = 30 \times 1 = 30 \text{ Coulombs}$$

**66. (b)**

$$\text{Here, } r = \frac{\sqrt{1^2 + 1^2}}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = 0.707 \text{ m}$$

$\therefore$  Electric potential at point O is

$$\begin{aligned} V &= \frac{1}{4\pi \epsilon_0} \left[ \frac{Q_1}{r} + \frac{Q_2}{r} + \frac{Q_3}{r} + \frac{Q_4}{r} \right] \\ &= \frac{9 \times 10^9}{0.707} (0.01 - 0.02 + 0.03 + 0.02) \times 10^{-6} \\ &= \frac{9 \times 10^9 \times 0.04 \times 10^{-6}}{0.707} = \frac{9 \times 4 \times 10}{1/\sqrt{2}} = 360\sqrt{2} \\ &= 509.116 \text{ V} \approx 509.2 \text{ V} \end{aligned}$$

**67. (a)**

Since  $\oint \vec{E} \cdot d\vec{l} = 0$  for a conservative field, therefore it is applicable to only electrostatic field.

**68. (d)**

Electric potential at a distance  $r$  from the electric dipole is

$$V = \frac{Ql \cos \theta}{4\pi \epsilon_0 r^2} = \frac{1}{4\pi \epsilon_0} \times (Ql) \times \frac{1}{r^2} \times \cos \theta$$

$$\therefore V \propto Ql \propto \text{dipole moment}$$

$$\propto \frac{1}{r^2} \propto \cos \theta \propto l$$